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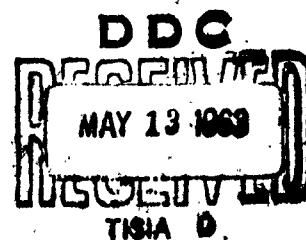
DISSIPATION EFFECTS CONTROLLING THE ELECTRICAL MORPHOLOGY OF THE MESOSPHERE

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PREFACE

A portion of the earth's atmosphere that has received relatively
little investigation is the mesosphere, lying between 20 and 80 km.
Knowledge about a variety of electrical properties and phenomena of
this region of the atmosphere is especially scanty. The mesosphere
is strongly responsive to a variety of excitation effects, including
bombardment by certain classes of solar protons and by solar ultra-
violet and x-radiations. These effects can produce notable variation
in the electrical parameters of the mesosphere.

A previous RAND Memorandum, RM-3230-PR, Effects of Cosmic
Radiation on the Extremely Low Frequency Properties of the Mesosphere,
examined some electrical properties of the mesosphere that are import-
ant to electromagnetic phenomena on the earth — micropulsations,
extra-low-frequency noise, and hydromagnetic waves. This Memorandum
investigates the electromagnetic structure of the mesosphere with
such geo-electromagnetic phenomena in mind. The analysis indicates
possible sources for certain stratification and ducting features of
the mesosphere that become prominent for slowly varying electrical
effects.

ABSTRACT

Electrical dissipation in the mesosphere is dominated by gas particle collisions. Hence, the electrical properties of this region are controlled by the density of the atmosphere. For low-frequency propagation, the electromagnetic properties of the mesosphere can be described in terms of variations with altitude of the local complex refractive index. For the simple condition of excitation of the mesosphere by a single kind of ionizing radiation, the complex refractive index should have exponential-scale variation with altitude of less than 12 km for the lower mesosphere. Sudden onset of more complex conditions of excitation may "trigger" the mesosphere into a state of "shelving" or ducted structure in its electrical properties. Depending on the details of the excitation, such ducting will contain regions of negative gradients in the altitude variation of the electrical properties, instead of merely the positive gradient with a 12-km upper limit.

ACKNOWLEDGEMENTS

CONTENTS

I. INTRODUCTION

The electrical properties of the lower ionosphere and of the mesosphere influence several geo-electromagnetic phenomena in the 1-10,000 c/s portion of the spectrum. Different atmospheric models show a variety of such phenomena: micropulsations (Bomke, 1962), cavity oscillations in the earth—ionosphere air space (Balser and Wagner, 1962), and distant propagation of lightning fields (Holzer and Deal, 1956; and Holzer, 1958). The models used include sharply bounded ionospheres (Schumann, 1952, 1957), special distributions such as Epstein layers (Epstein, 1930; and Schelkunoff, 1951), and two-layer models (Wait, 1960a). Exponential models of the variation of the electrical properties of the ionosphere with altitude have been used by Wait (1960b).

In a previous paper (Goldberg, 1962), the author discussed the extension of such modeling to three layers of the mesosphere: upper (60 km to 80 km), middle (40 km to 60 km), and lower (20 km to 40 km). The present paper furthers that discussion. It examines distinctive electromagnetic characteristics of the mesosphere, e.g., the conditions for negative altitude gradients of refractivity, those for relatively large gradients of refractivity, and those for abrupt transitions or "trigger situations."

II. ELECTRICAL PARAMETERS

We will use the complex index of refraction as a primary electrical parameter for describing the mesosphere's electrical structure. In terms of the electric number density n_e, and the electron collision frequency ν, the complex index is

$$M^2 = 1 - \frac{(\omega_p/\omega)^2}{1+(\nu/\omega)^2} - j \frac{(\omega_p/\omega)^2(\nu/\omega)}{1+(\nu/\omega)^2} \quad ,$$

where

M = the complex index of refraction,

ω_p = the angular plasma frequency, $(n_e e^2/m \, \epsilon_o)^{\frac{1}{2}}$,

e = the electronic charge,

m = the electronic mass,

ω = the angular frequency of the excitation field,

n_e = the electron-number density,

ϵ_o = the absolute permittivity of free space, and

j = $\sqrt{-1}$.

The refractive index is convenient for denoting electrical properties of the mesosphere in dimensionless values. (In terms of local values of the basic electrical properties of conductivity σ and relative dielectric constant ϵ_r, the complex index is given by the relationship

$$M^2 = \epsilon_r - j \frac{\sigma}{\omega \epsilon_o} \quad .$$

III. IONIZATION PRODUCTION MECHANISMS AND MODELS

Over most of the mesosphere, the electron density's variation with altitude can be represented by single exponentials or by composites of exponentials. The distribution of electron density with altitude depends (among other things) on the number and type of ionizing radiations present and on various processes of electron attachment, detachment, and recombination.

The exponential form of representation is developed here for very idealized conditions, and methods for treating the more complex cases are discussed briefly. The fact that even the more complex situations can be represented fairly simply by exponentials is a definite advantage.

For the simplest condition — a minimum of participating processes in equilibrium — the local electron density is given by the expression

$$n_e = \frac{q}{\beta} = \frac{q}{B_n(O_2)} \quad ,$$

where n_e is the electron density, q is the rate of production of electrons due to action of a single ionizing source, and the equality of β and $B_n(O_2)$ characterizes the loss of electrons due to attachment, with no other mechanism (such as detachment) being important. When mechanisms like detachment occur, the electron density distribution for much of the mesosphere can still be represented by single or combined exponential expressions.

As shown elsewhere (Goldberg, 1962), even a single, relatively ineffective source of ionization (such as cosmic radiation at night) produces refractivity values in the mesosphere that are significant for an electromagnetic excitation with slow time variations. In the daytime

and during solar storms, the electrical structure of the mesosphere is much more complicated, owing to the action of additional sources of ionization. These more complex conditions, which are excluded from this analysis, can be treated by combining similar models over different ranges of altitude.

For example, the electron density can be represented by

$$n_e = \left[\frac{Q}{(1+\lambda)(\alpha_d + \alpha_i \lambda)} \right]^{\frac{1}{2}} ,$$

where

Q = the total rate of production of electrons $q_1 + q_2 + q_3 + \ldots$ by primary ionizing mechanisms, 1, 2, 3, etc.,

λ = the ratio of negative ion to electron densities n^-/n_e,

α_d = the rate coefficient for dissociative recombination, and

α_i = the rate coefficient for Thompson three-body ionic re-combination.

To compare this expression for n_e with the expression for the simplest case ($n_e = q/\beta$), the above expression can be put in the form of

$$N = \left[N_{min} \right] \left[\frac{1}{Q^{\frac{1}{2}}} \right] \left[\frac{\beta}{(1+\lambda)^{\frac{1}{2}}(\alpha_d + \lambda \alpha_i)^{\frac{1}{2}}} \right] ,$$

where N_{min} is the value of N in the limiting case of $N = n_e = q/\beta$. This allows N to be expressed in the form of a product of three exponentials. The values of N for both the lower and upper mesosphere can be represented quite well by such a combined exponential, as indicated by comparison with the values of Moler (1960), Feier and Vice (1959), Reid (1961), and Landmark and Lied (1961), for various conditions

of the mesosphere. The following analysis of the altitude variations will be confined principally to the case of N_{min}, although it will make some use of composite exponential representations.

For the case of $n_e = q/\beta$, an exponential representation of the altitude variation of electron density n_e due to an ionization rate q can be obtained by writing

$$q = q_{oq} \exp\left[-(z-z_{oq})/Z_q\right] \quad,$$

where z_{oq} is the reference altitude at which $q = q_{oq}$ and where Z_q is the altitude scale-factor for q. Similarly, β can be written as

$$\beta = \beta_{o\beta} \exp\left[-(z-z_{o\beta})Z_\beta\right] \quad,$$

where $\beta_{o\beta}$ is the value of β at the reference altitude $z_{o\beta}$ and Z_β is the altitude scale-factor for the exponential variation of β.

These expressions for q and β give the general form for the exponential variation in altitude of the electron density:

$$n_e = n_{oq\beta} \exp\left(\frac{z_{oq}}{Z_q} - \frac{z_{o\beta}}{Z_\beta}\right) \times \exp z \left(\frac{1}{Z_\beta} - \frac{1}{Z_q}\right) \quad,$$

where $n_{oq\beta} = q_{oq}/\beta_{o\beta}$.

Also, in terms of composite variables to be used later, the electron density is

$$n_e = n_{eo} \exp (z_{on}/Z_{\beta q}) \times \exp (z/Z_{\beta q}) = n_{eo} \exp (z_{on}/Z_n) \times \exp (z/Z_n) \quad,$$

where n_{eo} is the value of the electron density at a common reference altitude, $z_{on} = z_{o\beta} = z_{oq}$; where $Z_{\beta q}$ is the composite altitude scale-factor,

$$\frac{1}{Z_{\beta q}} = \frac{1}{Z_\beta} - \frac{1}{Z_q} = \frac{1}{Z_n} \quad;$$

and where $Z_n = Z_{\beta q}$ when $z_{on} = z_{o\beta} = z_{oq}$.

Note that the electron-density altitude-variation factor, $Z_{\beta q}$, can be either positive or negative when the component scale-factors, Z_β and Z_q, are positive. This fact (and the possibility under various conditions in the mesosphere that the altitude scale-factors Z_q and Z_β can vary considerably in their magnitudes as well as in their signs) leads to some important consequences for the electrical properties of the mesosphere.

Variations in the altitude distributions of the ionization rate q or the loss characteristic β, as characterized by the scale factors Z_q and Z_β, produce a variation in the electron-density structure and in the electrical properties of the mesosphere.

We examine, first, a limiting case where q is uniform in altitude (i.e., $Z_q = \infty$), and, second, the variation of Z_β or Z_q in non-uniform ionization such as cosmic-ray activity at night.

Figure 1 shows how the electron-loss parameter β depends on altitude (due to such variations as air density and composition). With uniform ionization (i.e., $Z_q = \infty$), a doubling of the altitude scale-factor can cause a sixfold increase in electron density (for example, at an altitude of 40 km, between $Z_\beta = 5$ km and $Z_\beta = 10$ km).

Figure 2 shows another class of effects due to the variation of one altitude scale-factor relative to another. We see a non-uniform production of ionization with a single exponential of characteristic scale $Z_q = 6$ km. This value is selected as approximately equal to the atmospheric-density scale height; it thus represents ionization production that is proportional to atmospheric density and whose single source
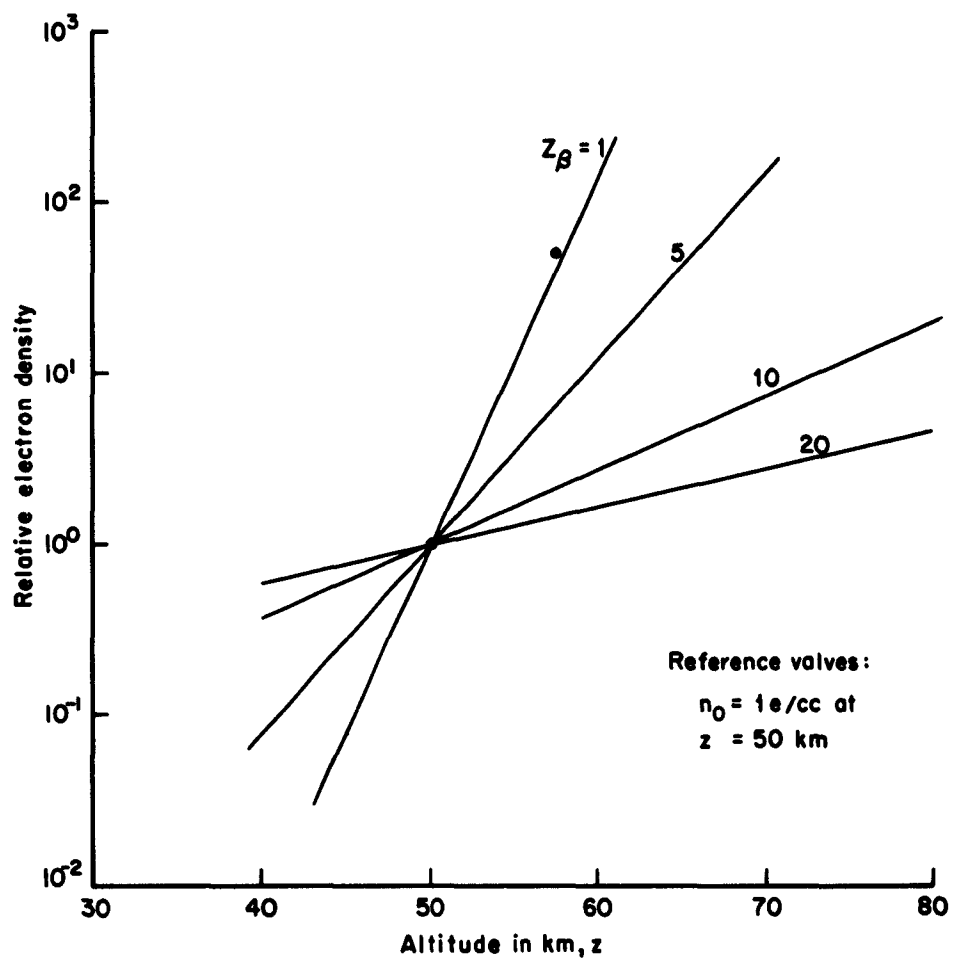
Fig. 1 — Dependence of electron density on variation
of electron — loss factor (or its equivalent, see text)
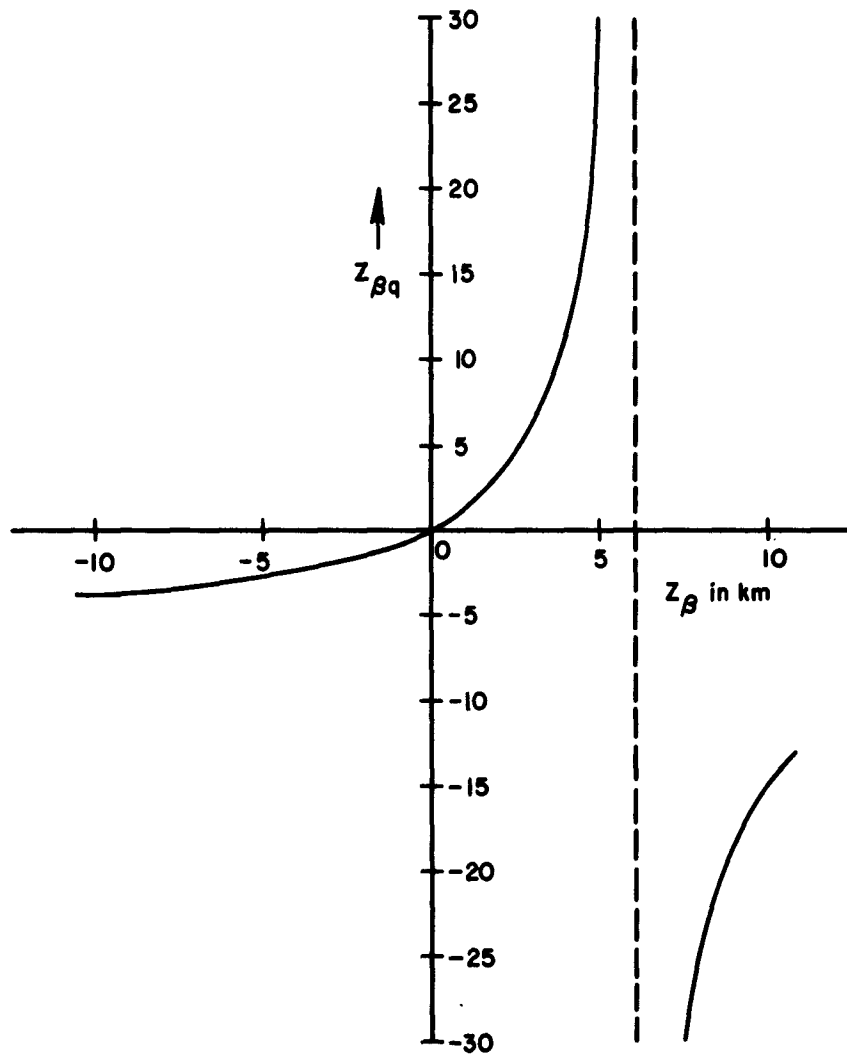for limiting case of electron — production factor $Z_q = \infty$

Fig. 2 — Dependence of electron—density altitude—
variation factor $Z_{\beta q}$ on electron loss factor Z_β
for case of electron —production altitude
factor $Z_q = 6$ km

is ultra-high-energy radiation, e.g., cosmic radiation at night. We
see that abrupt changes in the height-distribution of the electron
density can occur for changes in the atmosphere that produce quite small
variations of the altitude scale-factor for electron loss, z_β.

The $z_{\beta q}$ factor for the electron density has a symmetrical form
except for signs in the z_q and z_β components. Correspondingly,
therefore, "trigger conditions" or circumstances for abrupt changes
in the mesosphere's electron density can also exist for small variations
in z_q, i.e., in the altitude dependence of the ionizing mechanism. An
abrupt change in the electron density distribution of the mesosphere
can result, for example, from variations in the energy-distribution of
the incoming ionizing radiation. This effect is seen in excitation by
corpuscular radiation, treated by Reid (1961). He finds that high-intensity
fluxes of lower-energy solar particles produce an abrupt transition, both
in electron production at about 60 km in the middle mesosphere, and in
electron density at about 80 km in the upper mesosphere.

IV. EFFECTS ON THE ELECTRICAL MORPHOLOGY OF THE MESOSPHERE

Some noteworthy consequences for the mesosphere's electrical properties follow from the above considerations. They can be seen by examining the complex index of refraction, which, for most of the mesospheric region under consideration, can be approximated by the expression

$$M = \left[\omega_p / (\nu \omega)^{\frac{1}{2}} \right] \exp(-j\pi/4) \quad ,$$

as indicated in Goldberg (1962).

In terms of the electron density, with

$$\omega_p^2 = 3180 \, n_e = K n_e \, ,$$

M then takes the form

$$M = (K/\omega)^{\frac{1}{2}} (n_e/\nu)^{\frac{1}{2}} \exp(-j\pi/4) \quad ,$$

Using the exponential expression for the electron density variation with altitude,

$$n_e = n_{eo} \exp(z_o/Z_{q\beta}) \exp(z/Z_{q\beta}) \quad ,$$

and an exponential model of the collision frequency based on the values of Nicolet (1959),

$$\nu = \nu_{o\nu} \exp(z_{o\nu}/Z_\nu) \exp(-z/Z_\nu) \quad ,$$

we see that

$$M = (K/\omega)^{\frac{1}{2}} (n_{oq\beta}/\nu_{o\nu})^{\frac{1}{2}} \left[\exp(z_{on}/Z_{q\beta} - z_{o\nu}/Z_\nu) \right]^{\frac{1}{2}} \left\{ \exp \left[z(1/Z_{q\beta} + 1/Z_\nu) \right] \right\}^{\frac{1}{2}} \exp(-j\pi/4)$$

From this expression we can obtain a composite altitude scale-factor

Z_M given by

$$Z_M^{-1} = (2Z_{nq\beta})^{-1} = (2Z_{q\beta})^{-1} + (2Z_\nu)^{-1} = (2Z_q)^{-1} - (2Z_\beta)^{-1} + (2Z_\nu)^{-1} \; ;$$

Z_M characterizes the rate at which the mesospheric index of refraction M varies with altitude, since

$$M \simeq (K/\omega)^{\frac{1}{2}} \, (N_{eo}/N_{o\nu})^{\frac{1}{2}} \, \exp(z/Z_M) \, \exp(-j\pi/4) \quad ,$$

where

$$N_{o\nu} = \nu_{o\nu} \bigg/ \Big[\exp(z_{on}/Z_{\beta q} - z_{o\nu}/Z_\nu) \Big] \quad .$$

One physically significant condition is the limiting value of Z_M. Thus when electron production varies more slowly in altitude than does electron loss (or its composite equivalent in the expression above), Z_M in

$$\frac{1}{Z_M} = \frac{1}{2}\left(\frac{1}{Z_q} - \frac{1}{Z_\beta} + \frac{1}{Z_\nu}\right)$$

must always be less than $2Z_\nu$, which is about $2 \times 6.1 = 12.2$ km. The electrical dissipative process in the atmosphere (characterized by ν) not only provides local damping to electromagnetic effects, but also limits the rate at which such effects can vary with altitude. Because, in these regions of the atmosphere, ν depends principally on the atmospheric density, variations in the limiting value of the local refractive index will depend principally on variations of the atmospheric density.

Ion production by varying mechanisms and under differing atmospheric conditions can cause the mesosphere's electrical properties to have many variations with altitude, some with limiting conditions. Table 1

Table 1

ALTITUDE SCALE-FACTORS FOR DIFFERING CONDITIONS OF IONIZATION IN THE MESOSPHERE

Case	Reference	z_n (km)	z_M (km)
Daytime; 60-70 km	Fejer and Vice	8.2	7.2
Daytime; 70-80 km	Fejer and Vice	13.8	8.2
Nighttime; 65-80 km; cosmic radiation	Moler	5.5	5.6
Daytime; 50-70 km; cosmic radiation and Lyman α radiation	Moler	4.2	4.8
Daytime; 30-60 km; solar protons	Reid	4.1	4.8
Nighttime; 70-90 km; solar protons	Reid	-10.1	3.0
Summer day; 55-65 km	Landmark and Lied	5.2	5.4
Summer day; 65-70 km	Landmark and Lied	-3.5	-17.1

summarizes some of the states of ionization that have been treated in the literature. For the electron-density factor, Z_n, and the refractivity parameter, Z_M, it gives values computed for those portions of the n_e altitude profile that can be represented approximately in terms of a single exponential characteristic.

Figure 3 shows the altitude scaling-factor for mesosphere refractivity, Z_M. In this case, the collision-frequency altitude scaling-factor Z_ν is fixed at 6 km.

Note that the mesosphere can have three distinctive conditions of Z_n. These are:

1. Electron density increasing with altitude (upper right in Fig. 3). In this condition, the refractive index increases with altitude but does not vary more slowly than $Z_M = 12$ km.

2. Negative variations of electron density with altitude, where Z_n ranges from zero to -6 km (lower left in Fig. 3). The refractivity rapidly falls off with increasing altitude for values of Z_n lying between zero and 2 km. For $Z_n > 3$ km, the refractivity changes slowly with altitude, with Z_M varying from -12 km (for $Z_n = -3$ km) to infinity (for $Z_n = -6$ km).

3. Electron density falling off with altitude at a rate of $Z_n > -6$ km (upper left in Fig. 3). This leads to gradual upward increases of refractivity.

Variations with time in the excitation mechanism producing q (e.g., variations in flux or intensity of cosmic radiation, solar proton bombardment, or other processes affecting the state of ionization) will cause corresponding fluctuations in Z_q. This will lead to time variation
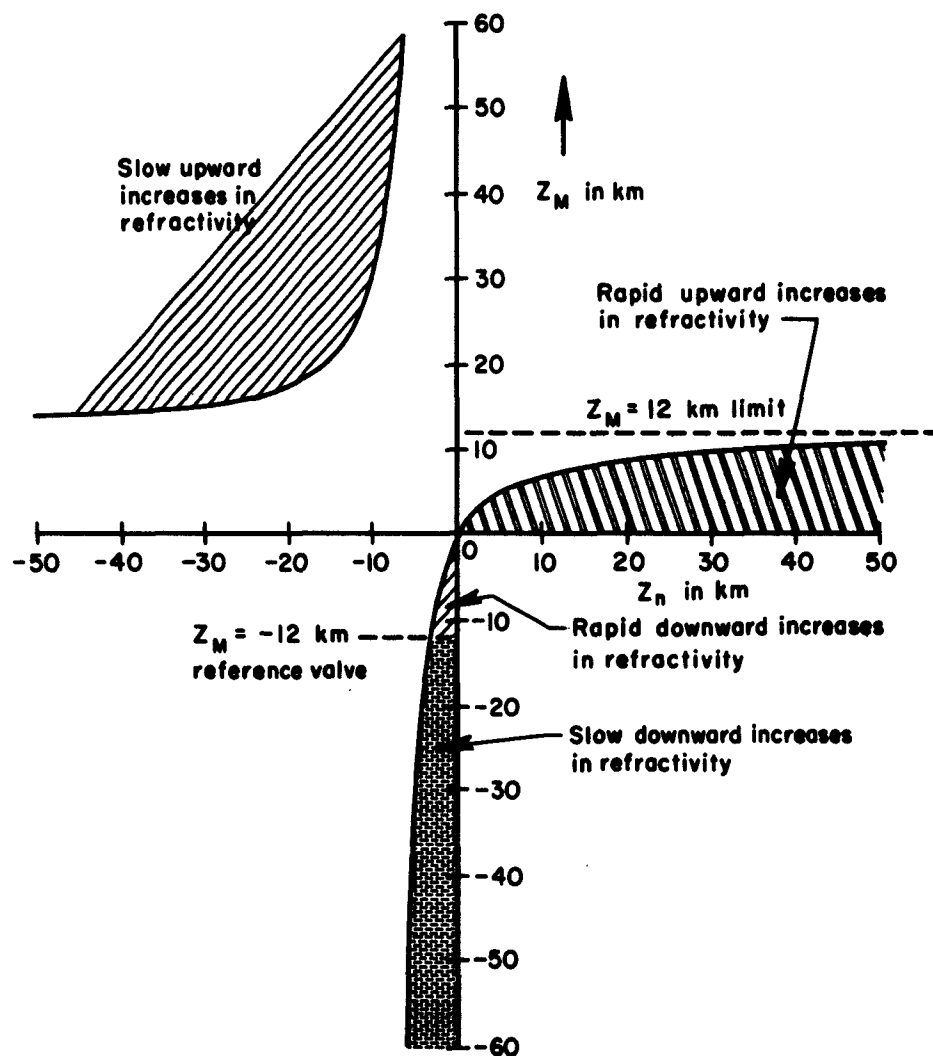
Fig. 3 — Four conditions of Z_M (the refractivity scale structure) in the mesosphere for various conditions of variation of Z_n — electron density with altitude (case of collision — frequency altitude variation Z_ν = 6 km)

of the electron density in the mesosphere and in the structure factors Z_n or $Z_{\beta q}$, resulting in a "modulation" of the electrical structure factor Z_M.

The variations can occur for any of the four conditions of mesospheric ionization shown in Fig. 3. However, the most sensitive atmospheric conditions (causing great responsiveness to changes in the modes of excitation) occur when the atmosphere is in those states summarized in Table 2.

Whether or not the value of Z_q varies considerably with altitude depends on the flux and energy distribution of the ionizing radiations. The mesosphere's electrical morphology can then be described in terms of segments of exponential variations of the parameter M. Under complex conditions, the varied electrical structuring depicted in the Z_M/Z_n diagram can occur simultaneously at various altitudes. This could happen when the mesosphere is excited by highly enhanced ultraviolet or x-radiation or by solar particles during solar storms. The diagram indicates that, at that time, shelves or strata could be a prominent mesospheric structure.

Table 2

CONDITIONS OF MESOSPHERE FOR SENSITIVE RESPONSE
IN ELECTRICAL STRUCTURE M BY VARIATION
OF IONIZATION DISTRIBUTION WITH ALTITUDE

Condition	Altitude Variation of Electron Density	Mesosphere Response
1	$0 < Z_n < 6$ km	M increases rapidly with altitude for small increases of Z_n and is positive
2	$-2 < Z_n < 0$ km	M decreases rapidly with altitude for small decreases in Z_n and is negative
3	$-6 < Z_n < -3$ km	M becomes 'diffuse downwards' (i.e., Z_M is large and negative), increasing slowly with decreasing altitude; large positive gradient changes in M occur for small decreases in Z_n
4	$-15 < Z_n < -6$ km	M becomes 'diffuse upwards' (i.e., Z_M is large and positive), increasing slowly with increasing altitude; large negative gradient changes in M occur for small decreases in Z_n

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